

LETTER TO THE EDITORS

FURTHER REMARKS ON AXISYMMETRIC BOUNDARY LAYER OVER A CIRCULAR CYLINDER

IN OUR recent article [1] we defined a function $g(\sigma)$ for which three different series representations were given—equations (54), (55) and (59). We later discovered that this function also possesses an integral representation

$$g(\sigma) = - \int_0^\sigma \frac{\log(1+r)}{r} dr.$$

The above integral is related to $S_2(X)$, $L_2(X)$ and $Rl(X)$ which are respectively Spence's integral for $n = 2$ [2], Euler's dilogarithm [3] and Powel's radiation integral [4]. These relations are as follows

$$g(\sigma) = S_2(1 + \sigma) = -Rl(1 + \sigma) = L_2(-\sigma)$$

where,

$$S_2(X) = -Rl(X) = - \int_1^X \frac{\log r}{r-1} dr$$

and,

$$L_2(X) = - \int_0^X \frac{\log(1-r)}{r} dr.$$

Powel [4] tabulates $Rl(X)$ for $X = 0-6$, corresponding to values of $\sigma = -1$ to 5. Series (55) with five terms or less will produce errors less than 10^{-6} for values of $\sigma > 5$. Hence there seems to be little need for extending Powel's tables to larger values of X .

The transformed series (59) was stated to be asymptotic with zero radius of convergence. The series has instead unit radius of convergence and it is therefore convergent for all positive values of the Prandtl number. Although (59) is more rapidly convergent than (54) for $\sigma < 1$, it is very sluggish compared to (55) for values of σ much larger than unity.

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REFERENCES

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